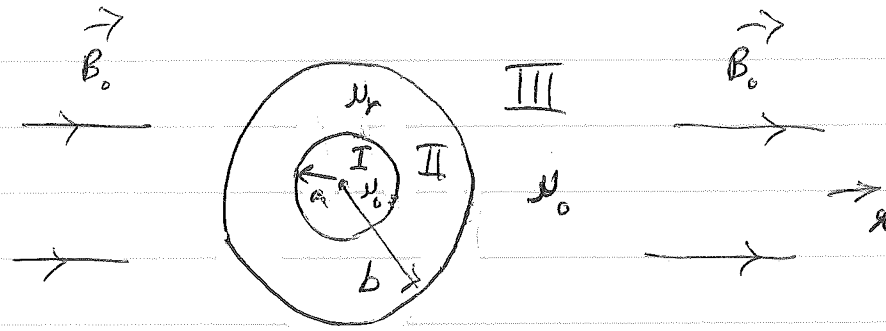
PHYC 511
Spring 2019

Problem Session 6 02/20/2019

(1) Problem 5.14, Jackson.

(2) Problem 5.19, part (a), Jackson.

(1) This problem is best suited by the means of the magnetic scalar potential as $\vec{J}=0$.



In region III:

$$H_o = \frac{B_o}{\mu_o} \Rightarrow \Phi_M^{(o)} = -H_o \rho \cos\phi$$

Then:

$$\Phi_M^{III} = \Phi_M^{(o)} + \Phi_M^{(induced)} = \boxed{-H_o \rho \cos\phi + \frac{A}{\rho}\cos\phi}$$

In region II:

$$\boxed{\Phi_M^{II} = \left(\frac{B}{\rho} + C\rho\right)\cos\phi}$$

Finally, in region I:

$$\boxed{\Phi_M^{I} = D\rho\cos\phi}$$

Using the continuity of H_t and B_n at $r=b$ and $r=a$ results in:

$$\Phi_M^{I}(a) = \Phi_M^{II}(a) \quad , \quad \Phi_M^{II}(b) = \Phi_M^{III}(b)$$

$$\mu_o \frac{\partial \Phi_M^{I}}{\partial r}(r=a) = \mu_r \frac{\partial \Phi_M^{II}}{\partial r}(r=a) \quad , \quad \mu_r \frac{\partial \Phi_M^{II}}{\partial r}(r=b) = \mu_o \frac{\partial \Phi_M^{III}}{\partial r}(r=b)$$

These will allow us to find A, B, C, D.

(2) $$\begin{cases} \vec{M} = M_0 \hat{z} & 0 \leq z \leq L \,,\; 0 \leq \rho \leq a \\ \vec{M} = 0 & \text{elsewhere} \end{cases}$$

This implies that:

$$\rho_M = \vec{\nabla} \cdot \vec{M} = 0 \,,\quad \sigma_M = \vec{M} \cdot \hat{n} = \begin{cases} +M & z = L \,,\; 0 \leq \rho \leq a \\ -M & z = 0 \,,\; 0 \leq \rho \leq a \\ 0 & \text{elsewhere} \end{cases}$$

Then:

$$\Phi_M(\vec{x}) = \frac{1}{4\pi} \int \frac{\sigma_M \, da'}{|\vec{x} - \vec{x}'|} = \frac{-M}{\cancel{4\pi}_2} \int_0^a \frac{\cancel{2\pi} \rho' d\rho'}{\sqrt{\rho'^2 + z^2}} + \frac{M}{\cancel{4\pi}_2} \int_0^a \frac{\cancel{2\pi} \rho' d\rho'}{\sqrt{\rho'^2 + (z-L)^2}}$$

Now:

$$\Phi_M(0,0,z) = -\frac{M}{4} \int_0^a \frac{d(\rho'^2 + z^2)}{\sqrt{\rho'^2 + z^2}} + \frac{M}{4} \int \frac{d(\rho'^2 + (z-L)^2)}{\sqrt{\rho'^2 + (z-L)^2}} =$$

$$-\frac{M}{4} \left[2(\rho'^2 + z^2)^{\frac{1}{2}} - 2(\rho'^2 + (z-L)^2)^{\frac{1}{2}} \right] \Bigg|_0^a = -\frac{M}{2} \left[\sqrt{a^2 + z^2} - |z| - \right.$$

$$\left. \sqrt{a^2 + (z-L)^2} - |L - z| \right]$$

From symmetry, $\vec{H}$ and $\vec{B}$ are along $\hat{z}$ on the z axis. Thus:

$$\vec{H} = H_z \hat{z} = -\frac{\partial \Phi_M}{\partial z} \hat{z}$$

Inside, $0 \leq z \leq L$, we have (note that $\vec{B} = \mu_0 (\vec{H} + \vec{M})$):

$$\boxed{\vec{H} = \frac{\vec{M}}{2} \left[\frac{z}{\sqrt{a^2 + z^2}} - \frac{(L-z)}{\sqrt{a^2 + (L-z)^2}} - 2 \right] \,,\quad \vec{B} = \mu_0 \frac{\vec{M}}{2} \left[\frac{z}{\sqrt{a^2 + z^2}} - \frac{L-z}{\sqrt{a^2 + (L-z)^2}} \right]}$$

Outside, $z>L$ or $z<0$, we have:

$$\boxed{\vec{H} = \frac{\vec{M}}{2}\left[\frac{z}{\sqrt{a^2+z^2}} - \frac{L-z}{\sqrt{a^2+(L-z)^2}}\right], \quad \vec{B} = \frac{\mu_0 \vec{M}}{2}\left[\frac{z}{\sqrt{a^2+z^2}} - \frac{L-z}{\sqrt{a^2+(L-z)^2}}\right]}$$

We see that $\vec{B}$ has the same expression both inside and outside. This is not surprising as B_n is continuous at the interface between two media, and here $\vec{B}$ is in the normal direction (i.e., $\hat{z}$) at the two faces of the cylinder.